SATELLITE IMAGE BLOCK ADJUSTMENT SIMULATIONS WITH PHYSICAL AND RPC CAMERA MODELS

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ABSTRACT

Satellite images can be block adjusted with either physical camera models or with replacement camera models. The rational polynomial coefficient (RPC) model is an example of a replacement camera model. Adjustment parameters for physical models typically include position, attitude, and interior orientation. Replacement models can replicate exterior orientation changes with translation and rotation of object coordinates. Replacement models with adjustable biases in image space can be used for block adjustment if the camera field of view is small and orientation errors are small. Tests of block adjusting actual imagery with physical and replacement models are necessarily limited in the number of images, ground control point configurations, and a-priori accuracy values employed. Simulations can explore the accuracy effects of block adjustment with physical and replacement camera models across a broader range of conditions. Simulation results will be presented showing the effects of varying a-priori accuracy and ground control point (GCP) configuration on the accuracy of block adjustments with physical and RPC camera models.

INTRODUCTION

Background

High-resolution satellite imaging systems differ in several important respects from historic, airborne film cameras for which photogrammetric theory and practice is well established. Aerial cameras have a wide field of view (FOV) of about 90 degrees. Recent high-resolution satellite cameras, such as IKONOS and QuickBird, have a field of view of about one-degree--nearly two orders of magnitude smaller. Although increasing use of airborne GPS and inertial measurement unit (IMU) systems is improving a-priori knowledge of aerial camera orientation, early aerial cameras had little or no a-priori knowledge of exposure station position and orientation. The interior orientation of metric, digital camera systems, whether airborne or satellite, can be accurately calibrated and so do not require solution during block adjustment. When compared to aerial cameras, recent high-resolution satellite imaging systems have a much narrower field of view, much better a-priori exterior orientation, and a known interior orientation.

Today's high-resolution satellite cameras exhibit different engineering designs requiring different mathematical descriptions. The increasing variety and complexity of physical camera models presents an implementation challenge to software developers and data users. A generic camera model would simplify software development and facilitate applications using new camera designs. Several photogrammetric software packages, such as SOCET SET™ and ImageStation™, can replace physical models of frame, satellite, or other cameras, with polynomial or rational polynomial models. The RPC camera model is an example of the class of replacement camera models variously called fast camera models, generic camera models, rational function models, etc. RPC models are available for imagery from the IKONOS, QuickBird, OrbImage, and other satellite systems. It was initially thought that RPC descriptions of image geometry did not support block adjustment. [Grodecki & Dial, 2003] showed that block adjustment of IKONOS imagery was possible with RPC models and mentioned in passing that RPC models should be applicable to any camera system with a narrow field of view and strong a-priori information. In this paper we wish to substantiate that observation. Published tests of RPC block adjustment include [Fraser & Hanley, 2004] and [Ager, 2003]

Objective

The objective of this paper is to explore the parameter estimation of a narrow field camera and its implications on replacement camera models such as the rational polynomial coefficient (RPC) model. For narrow field cameras, we show that translations and rotations about axes perpendicular to the line of sight (LOS) are not independently observable and that ground control observations do not improve available a-priori accuracy of focal length, flying height, or rotation about the LOS. These results are contrary to wide field of view cameras for which translations, rotations, and scale can be independently determined from ground observations. Further, we will show that block adjustment by image space adjustment of RPC models is generally possible for systems with a narrow field of view, know interior orientation, and strong a-priori exterior orientation.

METHODOLOGY

Parameter estimation with narrow field cameras might be explored in various ways. It might be possible to prepare a purely mathematical analysis. We will present a simple mathematical discussion to motivate our approach. One could form the observation equations and analyze the solution by covariance analysis or singular value decomposition (SVD). We have chosen to use numeric, Monte-Carlo simulation to illustrate the block adjustment of narrow field cameras with physical and RPC camera models. The physical model studied will be the central perspective model of a frame camera, albeit, a frame camera with a narrow field of view, known interior orientation (apart from scale), and strong a-priori.

Mathematical Motivation

Experience with aerial cameras shows that camera position and orientation can be independently determined by ground observations. Statements that camera position and angle cannot be independently determined for narrow field cameras might be viewed suspiciously. In [Grodecki & Dial, 2003], we used the figure shown below to demonstrate the equivalence of small translations and rotations for narrow field cameras.

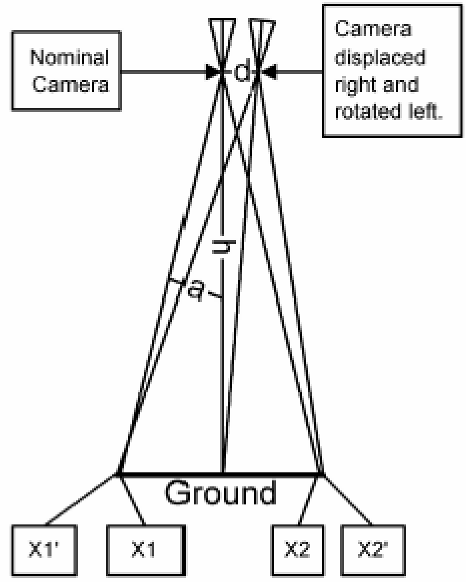


Figure 1. Effect of roll and cross-track errors.
a = half-angle of the camera field of view; for Ikonos, a = 28.53′.
h = orbital height; for Ikonos, h = 680 km.
r = camera roll angle = 2 arc-seconds.
d = equivalent displacement = h tan(r) = 6.593466 m.
X1 = ground coordinate of the left edge of the nominal camera.
= −h tan(a) = −5644.129609 m.
X1′ = ground coordinate of the left edge of the displaced and rotated camera.
= d − h tan(a + r) = −5644.130063 m.
X2 = ground coordinate of the right edge of the nominal camera.
= h tan(a) = 5644.129609 m.
X2′ = ground coordinate of the right edge of the displaced and rotated camera.
= d + h tan(a − r) = 5644.129155 m.
X1 − X1′ = X2 − X2′ = 0.000454 m.

Figure l. Effect of roll and cross-track errors

The figure shows that two camera orientations, one with a nominal, straight down, orientation and the other rolled two arc-seconds to the left and translated 6.593 meters to the right, differ in their field width by less than 0.5 mm. The small difference between translation and rotation is a consequence of the small angle approximation.

The mapping from ground coordinates to image coordinates involves translations from ground coordinates to the camera perspective center, angular rotations involving sines and cosines in object space, and the perspective equation with its tangent relationship inside the camera. The trigonometric functions can be represented by Taylor series expansions.

$\sin(x) = x - x^3/3! \ldots$

$\cos(x) = 1 - x^2/2! \ldots$

$\tan(x) = x + x^3/3 \ldots$

The largest deviation from the small angle approximation involves terms of order x^2 where x is an angle in radians. The largest distance that might multiply the deviation is the camera height, so deviations from second-order terms are of the order of x^2 h. If all angles (and angle-equivalent translations) are known to better than 10^{-4} radian (about 20 arc-seconds) and the camera height is less than 10^6 m, then terms of size x^2 and higher cause deviations of less than 0.01 meter. Commercial satellite imaging systems fly below 1000 km and provide a-priori orientation to

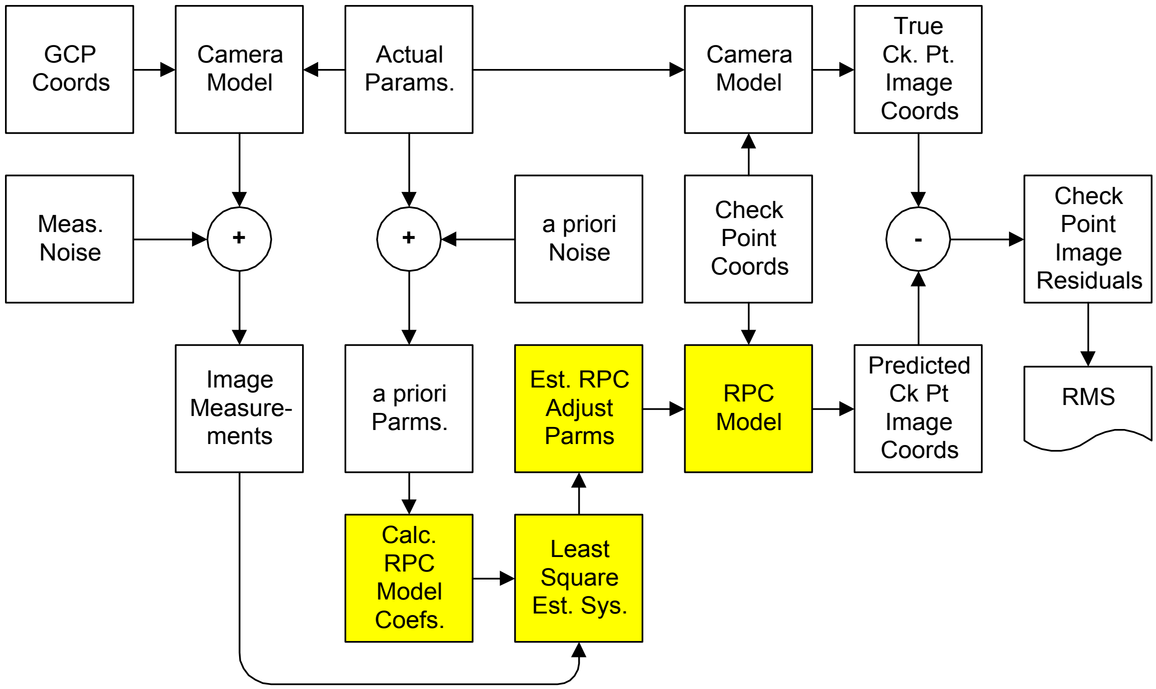


Figure 3. Simulated block adjustment with RPC camera model tested with checkpoints.

The simulation process for RPC block adjustment, shown in figure 3, is identical to the physical camera model process in figure 2 except for the highlighted boxes that will now be explained. The a-priori physical camera model parameters are used to calculate the RPC coefficients. The least square process estimates the RPC adjustment parameters. The check point image coordinates are now predicted from the RPC model as adjusted by the least square process.

Details of each of these steps will now be given for the physical and RPC simulations.

Physical Camera Model

For a central perspective camera, image coordinates (Xi, Yi) in the focal plane are related to object coordinates (Xc, Yc, Zc) in a camera coordinate system by

$$\begin{bmatrix} Xi \\ Yi \end{bmatrix} = f \begin{bmatrix} Xc/Zc \\ Yc/Zc \end{bmatrix} \tag{1}$$

The camera coordinate system has Zc along the optic axis and Xc and Yc perpendicular to the optic axis. Camera coordinates are related to a ground coordinate system defined by the exterior orientation equation:

$$\begin{bmatrix} Xc \\ Yc \\ Zc \end{bmatrix} = Rz(Az)Ry(Ay)Rx(Ax)(P_G - P_O). \tag{2}$$

Here *Rx*( ), *Ry*( ), and *Rz*( ) denote 3 x 3 rotation matrices about their respective axes. We designate angles by Az, Ay, and Ax for mnemonic value where ω, ϕ, and κ might have been more conventional. The ground coordinate is P_G = (Xg, Yg, Zg) and the camera perspective center is at P_0 = (Xo, Yo, Zo) in the ground coordinate system.

Parameters of the physical camera model used in the estimation process will be the perspective center position, the angular rotations, and the focal length so that the estimation parameter vector is (Xo, Yo, Zo, Ax, Ay, Az, f).

RPC Camera Model

The RPC equation for image coordinates is the ratio of two polynomials of the object space coordinate. For central perspective cameras, image coordinates are the ratio of two linear functions of object coordinates:

$$Xi = \frac{C_{X1} + C_{X2}\,X + C_{X3}\,Y + C_{X4}\,Z}{D_{X1} + D_{X2}\,X + D_{X3}\,Y + D_{X4}\,Z}$$

$$Yi = \frac{C_{Y1} + C_{Y2}\,X + C_{Y3}\,Y + C_{Y4}\,Z}{D_{Y1} + D_{Y2}\,X + D_{Y3}\,Y + D_{Y4}\,Z} \qquad (3)$$

where

(Xi, Yi) = focal plane coordinate,

(X, Y, Z) = ground coordinate, and

$\{C_{Xi}, D_{Xi}, C_{Yi}, Dyi\}$ = RPC coefficients defining the camera object-image relationship.

Commonly, RPC equations have terms up to cubic in X, Y, and Z for a total of 80 coefficients. For a central perspective frame camera, the 16 coefficients enumerated above can be computed analytically from the physical camera parameters (Xo, Yo, Zo, Ax, Ay, Az, f). Higher order RPC equations can model distortions in optics, scanning, or coordinate representations.

Block adjustment using only replacement camera models, such as the RPC model, seems difficult because of the 80 coefficients in even a cubic model. However, if the interior orientation of the camera is know and accurately represented by the replacement model, then we need only transform the ground coordinate input to the replacement model. To show this, rewrite equation (3) in vector notation:

$$(Xi, Yi) = \mathbf{R}(P_G) \qquad (4)$$

where **R** is the replacement camera model function and, as before, *(Xi, Yi)* is an image coordinate and P_G is a ground coordinate. The object-image relationship for the same camera with a different exterior orientation could be written

$$(Xi, Yi) = \mathbf{R}(Rz(Az)\,Ry(Ay)\,Rx(Ax)\,(P_G - P_O)) \qquad (5)$$

where P_O is the offset and *Rx(Ax) Ry(Ay) Rz(Az)* is the rotation from one ground coordinate system to another to adjust the exterior orientation of the camera. Thus replacement models that accurately represent the interior orientation of a camera can be exactly adjusted to change exterior orientation.

If the offsets and rotations are small, then we can expand (5) in a Taylor series, keeping only the bias term:

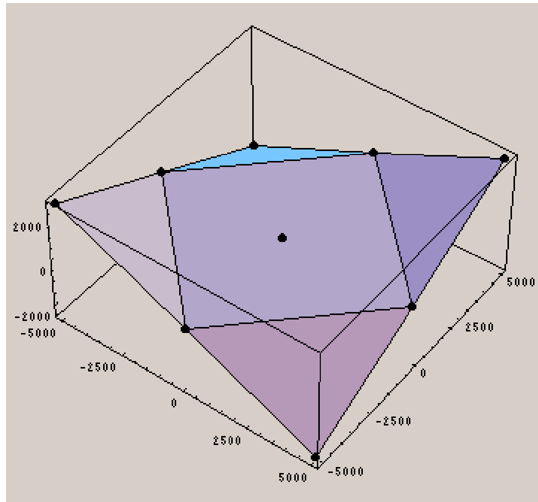
$$(Xi, Yi) = \mathbf{R}(P_G) + \begin{bmatrix} Bx \\ By \end{bmatrix} \qquad (6)$$

In this bias-compensated RPC model, *Bx* absorbs the first-order effect of parameters *Xo* and *Ay* that move the image along the x-axis and By similarly absorbs the first-order effects of *Yo* and *Ax*. Higher order terms are not needed if the field of view is narrow and the parameter errors are small.

Solution for the RPC bias terms in (6) is particularly easy. Simply set the biases to the average image error to remove the effect of exterior orientation errors.

Ground Control Configurations

Ground control point configurations are considered with 1, 4, and 9 GCP. The one GCP is located at X = Y = Z = 0. The GCP configuration with four GCP is weak with GCP at X = ± 1 km, Y = ± 1 km, and Z = 0. The configuration with nine GCP is strong with GCP every 2.5 km for ± 5 km in X and Y and distributed ± 2.5 km in Z to form a topographic saddle as illustrated below.

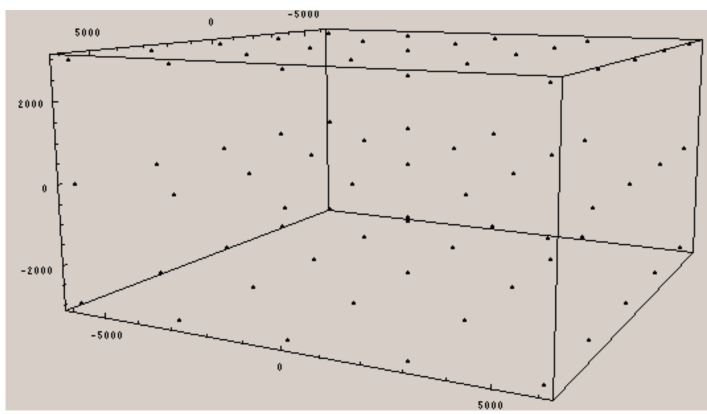


9 GCP arranged ±5 km in X and Y and ±2.5 km in Z

Check Points

Solution accuracy is evaluated with checkpoints. The check point image coordinate is predicted with the adjusted camera model. The actual image coordinate is calculated with the camera model using actual parameters. The RMSr of difference between predicted and actual image coordinates evaluates solution accuracy.

The checkpoints are at 3-km increments spanning a ±6 km range of the X and Y axes and 2.5 km increments spanning a ±2.5 km range in Z. This arrangement provides a thorough test of adjustment accuracy throughout the volume viewed by the camera as shown in the figure below.



75 Checkpoints arranged ± 6 km in X and Y and ±2.5 km in Z

Estimation System

Least-squares estimation solves for the optimum parameter vector ***X*** (not to be confused with x-axis coordinates) that minimizes the sum-square error of the measurement and a-priori residuals:

$$J = \sum_{i=1}^{7} \left(\frac{X_i - X_{api}}{\sigma_{api}} \right)^2 + \sum_{i=1}^{N} \left(\frac{x_{mi} - x_i(X)}{\sigma_m} \right)^2 + \sum_{i=1}^{N} \left(\frac{y_{mi} - y_i(X)}{\sigma_m} \right)^2 \qquad (7)$$

where

J is the sum-square error cost function,
X is the parameter vector $X = (Xo, Yo, Zo, Ax, Ay, Az, f)$,
X_i is the i-th component of the parameter vector X
X_{api} is the a-priori estimate of the i-th component of X
σ_{api} is the a-priori sigma of the i-th component of X
N is the number of GCP (1, 4, or 9),

(x_{mi}, y_{mi}) is the measured image coordinate of the i-th GCP,

($x_i(X)$, $y_i(X)$) is the image coordinate of the i-th GCP calculated using parameter vector X, and

σ_m is the measurement sigma.

Solving for parameter vector X to minimize J in equation (6) follows standard least-squares procedures.

A brief least-square example may be instructive to the present study. Consider a measurement Y that is the sum of two random variables, X_1 and X_2, with some measurement error v, i.e., $Y = X_1 + X_2 + v$. We might have a-priori estimates of X_1 and X_2, e.g., $X_1 \sim N(0, \sigma_{ap})$ and $X_2 \sim N(0, \sigma_{ap})$ and characterize the measurement error by $N(0, \sigma_m)$ where $N(\mu, \sigma)$ designates a normal random variable with mean μ and standard deviation σ.

If the measurement is accurate with $\sigma_m << \sigma_{ap}$, then the uncertainty of X_1 and X_2 will be approximately $\sigma_{ap}/\sqrt{2}$ and the estimates will be highly correlated with R ~ -1. We liken this situation to the simultaneous estimation of translations and rotations about axes perpendicular to the line of sight. The image measurements provide information about the sum effect of the translation and rotation but little information about the individual values of translation and rotation. Evidence that measurements relate to the sum (or difference) of two parameters but do not distinguish between them is seen in high correlated (or anti-correlated) parameter estimates with scatter related to their a-priori uncertainties rather than the measurement accuracy.

Alternatively, if the measurement is of poor accuracy $\sigma_m >> \sigma_{ap}$, then the uncertainty of X_1 and X_2 will be remain approximately σ_{ap} and the estimates will be uncorrelated with R ~ 0. We liken this situation to the simultaneous estimation of focal length and height for a narrow field camera. The a-priori provides more accurate information about focal length and height than the image measurements. Evidence that measurements do not provide information on particular components of the parameter vector is seen in uncorrelated parameter estimates with a posteriori sigmas that are not reduced from their a-priori values.

Assumptions

To reduce the large ground distances, small image distances, and micro angular rotations to common units, we scale all parameters to meters in ground space as will be explained. The simulation values shown in the table below are generalized from current high-resolution satellites and not specific to any one system.

Parameter	Symbol	Value
Nominal camera height	h, Pz	500,000 m
Focal length	f	500,000 m
Field of view	θ	0.01
Measurement accuracy	σ_m	0.5 m
a-priori uncertainty	σ_{ap}	1, 2, 5, 10, 20, 50, or 100 m
Orientation	Ax, Ay, Az	0 (nadir)
Offset	Xo, Yo	0

Setting focal length f equal to the nominal height h results in distances in the focal plane being measured in ground units, i.e., moving a point 1 km on the ground produces a 1 km equivalent change in its image coordinates. In reality, focal plane dimensions are much smaller than ground dimensions because focal lengths are much shorter than the camera height, but this scaling of interior dimensions of the camera does not result in any loss of generality in the conclusions. The field of view remains small by virtue of the focal plane being small (±5 km) compared to the camera height (500 km).

If angles are measured in radians, then angular errors tend to be very small, non-intuitive numbers. Here we measure angles in meters ground displacement per meters of camera height. Example: a one micro-radian (1 uR) rotation about an axis perpendicular to a 500 km long line-of-sight results in 0.5 meter displacement on the ground; instead of saying 1 uR, we call this an 0.5 meter rotation.

Each of the 7 parameters in the state vector of the physical model should have its own a-priori uncertainty based on the accuracy of the particular systems determining position, angle, or calibrating focal length. For this case study, we simply set the a-priori sigma of all of the parameters to the same value, which ranges from one meter one-sigma to one hundred meters one-sigma. Setting all of the a-priori sigmas to the same value is possible because all of the parameters have the same dimensionality.

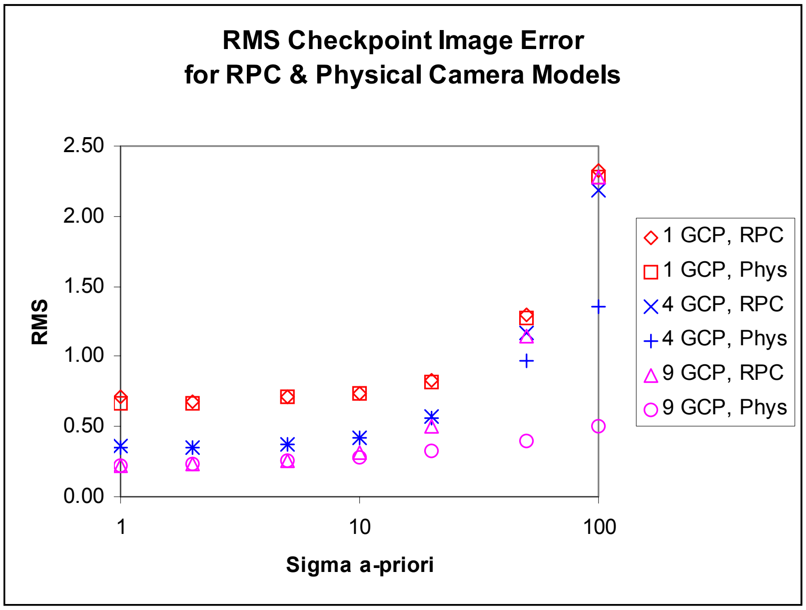
For the RPC model, we set the a-priori sigma of the bias parameters to $\sqrt{2}$ times σ_{ap}. This is because the physical model had σ_{ap} uncertainty in translations and σ_{ap} equivalent uncertainty in angles and in the RPC model each bias must absorb the effects of both a translation and a rotation.

RESULTS

The Monte Carlo simulation was run 250 times for each a-priori sigma value from one to 100 meters. The 7-parameter physical model was becoming numerically unstable at 100 meter a-priori, so larger a-priori uncertainties were not attempted. The paragraphs below examine the RMS error and parameter estimates from the simulation.

RMS Checkpoint Error

The RMS checkpoint errors for the 3 different GCP configurations are shown below for the RPC and physical camera model.



Improving the ground control reduces the RMS error. If the a-priori sigma is 10 meters or lower, performance of the RPC and physical model is indistinguishable. The physical model performs better than the RPC model if the ground control is strong and the a-priori is weak. If the ground control is weak or the a-priori is strong, then the RPC model performs just as well as the physical model. Adjustment of the RPC model did not have the numerical difficulties encountered adjusting the physical model.

Parameter estimates of the Physical model

We will examine the parameter estimates for the case of 9 GCP and an a-priori sigma of 5 meters. These results are typical of results for other GCP configurations and a-priori values.

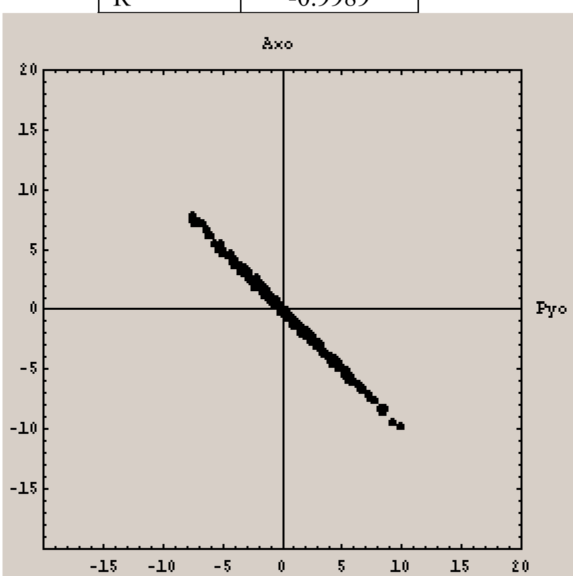
X-Offset vs. Y-Rotation. The table and scatter plot below show the x-offset and y-rotation adjustments of the physical model for the case of 9 GCP and a-priori sigmas of 5 meters.

Statistic	Xo	Ay
Mean	0.28	0.27
Std. Dev.	3.27	3.26
R	0.9988	

Estimates of the x-axis offset are seen to be completely correlated with rotations about the y-axis. The image measurements relate to the total effect of Ay and Xo and not to contribute information to their individual values.

Y-Offset vs. X-Rotation. The table and scatter plot below show the y-offset and x-rotation adjustments of the physical model for the case of 9 GCP and a-priori sigmas of 5 meters.

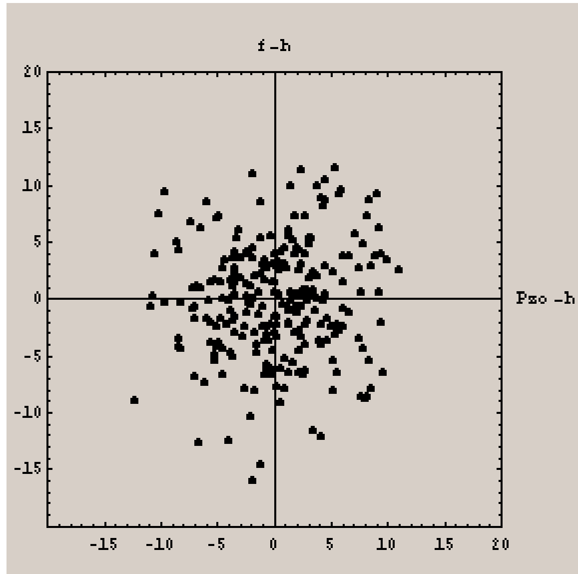
Statistic	Yo	Ax
Mean	0.07	-0.05
Std. Dev.	3.65	3.64
R	-0.9989	



Estimates of rotation about the x-axis are seen to be completely correlated with offset along the y-axis. Again, the measurements do not distinguish between the effects of Ax and Yo.

Z-Offset vs. Focal Length. The table and scatter plot below show the z-offset and focal length adjustments of the physical model for the case of 9 GCP and a-priori sigmas of 5 meters.

Statistic	Zo	f
Mean	500000	500000
Std. Dev.	4.72	4.99
R	0.07	



The a-priori sigmas of the z-offset and focal length parameters were 5 meters. It is seen that even with a strong GCP configuration that the estimation process did not significantly reduce the a-priori sigmas. Even though ground measurements bear on the combined effects of focal length and camera height on scale, the parameters are not correlated because the measurement was too weak to contribute significantly to the estimation of focal length or scale.

Rotation about the Optic Axis. Statistics for the physical model estimates of Az, rotation adjustment about the optic axis, are tabulated below for the case of 9 GCP and an a-priori sigma of 5.

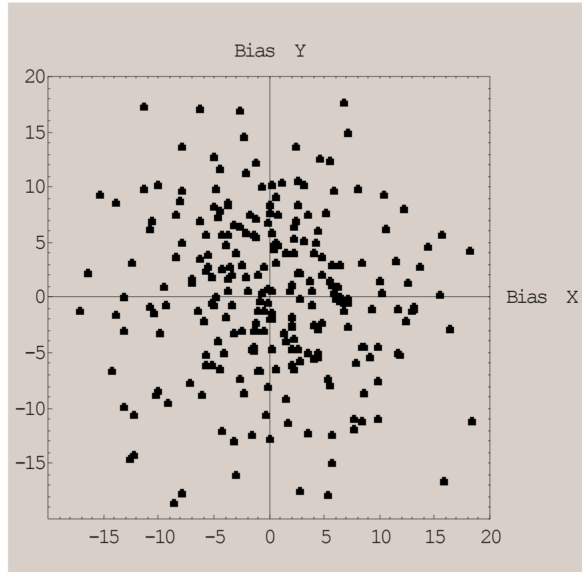
Statistic	Az
Mean	-0.46
Std. Dev.	4.56

Rotations about the optic axis do not have a significant impact on the image coordinates so the parameter estimates scatter according to their a-priori.

Parameter estimates of the RPC model

The table and scatter plot below show the x- and y-axis bias adjustments to the RPC model for the case of 9 GCP and a-priori sigmas of 5 meters.

Statistic	Bx	By
Mean	0.36	0.08
Std. Dev.	7.17	7.54
R	-0.05	



The a-priori sigmas of the RPC biases were 5√2 or 7.1 meters and the parameter estimates scatter by about this much. The parameter estimates appear uncorrelated. Unfortunately we do not have a covariance analysis to confirm that these parameters are accurately estimated with small a-posteriori uncertainty. The small RMS checkpoint errors demonstrate that the adjusted RPC model accurately describes the object-image relationship. No numeric difficulties were encountered estimation the RPC bias parameters.

CONCLUSION

Our objective was to explore the block adjustment of a narrow field camera similar to today's high-resolution, commercial imaging systems. Simultaneous solution for position and orientation of a narrow field camera is numerically unstable because multiple parameters have the nearly same effect on image positions. The higher order effects that might distinguish translations and rotations are too small to measure in a narrow field camera. Rotations about the optic axis cause insignificant changes in image position if those rotations are small. The focal length and height of a narrow field camera are not improved by ground observations if the a-priori estimates are good. A system satisfying these qualifiers was simulated with results confirming these statements.

We also considered a replacement camera model, such as the RPC model, instead of the physical model. If the interior orientation is known, then the replacement model with offsets and rotations of its object coordinates can adjust its exterior orientation without any loss of accuracy. Bias adjustments to image coordinates can be used instead for cameras with a narrow field of view. Comparisons showed that the bias-compensated RPC model was just as accurate as the physical model with sufficiently accurate a-priori. The physical model was more accurate than the RPC model only if both the ground control configuration is strong and the a-priori is weak.

REFERENCES

Ager, Tom (2003). Evaluation of the geometric accuracy of Ikonos imagery, SPIE 2003 AeroSense Conference, 21-25 April 2003, Orlando, Florida.

Fraser, Clive & Harry Hanley (2004). Bias-Compensated RPCs for Sensor Orientation of High-Resolution Satellite Imagery, *Proceedings of ASPRS Conference, Denver, CO*, unpaginated CD-ROM.

Grodecki, Jacek and Gene Dial (2003). Block Adjustment of High-Resolution Satellite Images Described by Rational Polynomials, *Photogrammetric Engineering & Remote Sensing*, 69(1):59-68, January 2003.