

# ACCURACY ANALYSIS OF RATIONAL POLYNOMIAL COEFFICIENTS FOR IKONOS IMAGERY

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## ABSTRACT

Rational polynomial coefficients (RPCs) supplied with IKONOS images provide a compact representation of the ground-to-image geometry, allowing end users to perform photogrammetric processing without requiring access to the physical camera model. While rational function models have largely been accepted by the photogrammetric community, they are still sometimes referred to as an 'approximate model' suggesting that they confer a lower degree of accuracy than the physical model.

In order to assess the accuracy with which rational polynomials can express the viewing geometry for IKONOS images, RPCs are compared with the IKONOS physical camera model for a number of different datasets. The effect of image length, terrain height variation and imaging geometry are considered in the comparison.

## INTRODUCTION

Photogrammetric tasks such as block adjustment, 3D feature extraction and orthorectification are accomplished by the use of a camera model expressing the relationship between object space (points on the ground) and image space (pixels in the image). For IKONOS Geo Ortho Kit images, this camera model is provided in the form of a pair of cubic rational polynomial functions expressing the ground-to-image geometry (Grodecki and Dial, 2001). Numerical values for the polynomial coefficients are provided as part of the image metadata.

While the rational polynomial camera model is simple for the end user to implement, determination of the model coefficients requires some calculation. With a pushbroom sensor such as IKONOS, there is no direct relationship between rational polynomial coefficients and the parameters of the physical camera model; therefore, the usual procedure for RPC determination involves using the physical camera model to generate a 3D grid of points, followed by least squares estimation of the coefficient values to fit this grid. This process has been described by several authors (Tao and Hu, 2001; Hartley and Saxena, 2000; Grodecki, 2001) for use with IKONOS and numerous other sensor types.

The quality of the least squares fit depends on a number of factors, including but not limited to: view geometry, satellite dynamics during image capture, image strip length, range of heights used in the 3D point grid, and the spacing of grid points. The influence of some of these factors was analyzed by Grodecki and Dial (2001); using simulated imaging scenarios, they computed a worst-case RPC fit error of 0.04 pixels for strip lengths up to 100 km.

In this paper, we examine in more detail the influence of image strip length and height range on the accuracy of IKONOS RPCs. Rather than simulated imaging scenarios, several real-world datasets are used, thus providing a basis for validation of the predicted results.

## IKONOS GEO IMAGES AND RPC ESTIMATION

The purpose of the tests described herein is to examine how well the rational polynomial camera model can represent the ground-to-image geometry for a typical IKONOS Geo Ortho Kit image. Before the test procedure is described, a brief overview of the Geo image product and RPC estimation procedure is given.

The minimum level of processing generally offered to commercial IKONOS customers is known as a Geo image product. The Geo product is created by projecting the original, uncorrected image onto an inflated ellipsoid and resampling it to a regularly-spaced pixel grid. The georectification process works as follows: first, the area of coverage for the Geo image is defined depending on the customer Area of Interest (AOI), and a rectification height

is chosen based on the average terrain elevation. For each pixel in the Geo image, the IKONOS physical camera model is used to project the pixel's longitude, latitude, and rectification height into sample and line in the uncorrected image. A pixel value is then interpolated from this location in the uncorrected image and stored in the Geo image.

To create a 'camera model' suitable for further photogrammetric processing of the Geo image, it is necessary to convey information about the line of sight vector at each pixel location. As mentioned in the previous section, this is done by computing RPCs for a 3D grid of points covering the image. To generate the 3D grid of points, a subset of points covering the Geo image (for example, every 200 pixels in line and sample) is used. Each pixel location in the Geo image corresponds to a specific location in ground space, defined by its longitude and latitude and the image rectification height. For this ground space location, the IKONOS physical camera model is used to determine the line of sight vector passing through the pixel. Latitude and longitude are then computed for a number of elevation values covering the expected terrain range. The line of sight vectors are computed for a large number of points in the Geo image, and the resultant 3D point grid is used to estimate the RPCs by the method of least squares.

To test the accuracy of RPC estimation for a typical image, several uncorrected IKONOS scenes have been obtained, along with their associated attitude and ephemeris information and the Field Angle Map (Grodecki and Dial, 2001) defining the interior orientation parameters of the IKONOS physical camera model. Table 1 provides a summary of the scenes used for the tests. The different scenes represent a range of different strip lengths and collection geometry.

**Table 1. Test scenes used for RPC accuracy assessment**

Scene	Collection Date	Strip Length [km]	Collection Azimuth [deg]	Collection Elevation [deg]	Scan Azimuth [deg]
<b>Thornton</b>	2001-Jun-09	19	343	68	0
<b>BigSpring</b>	2002-Mar-29	33	309	83	0
<b>SanDiego_1</b>	2000-Jan-30	61	21	61	180
<b>SanDiego_2</b>	2000-Jan-30	71	171	77	180

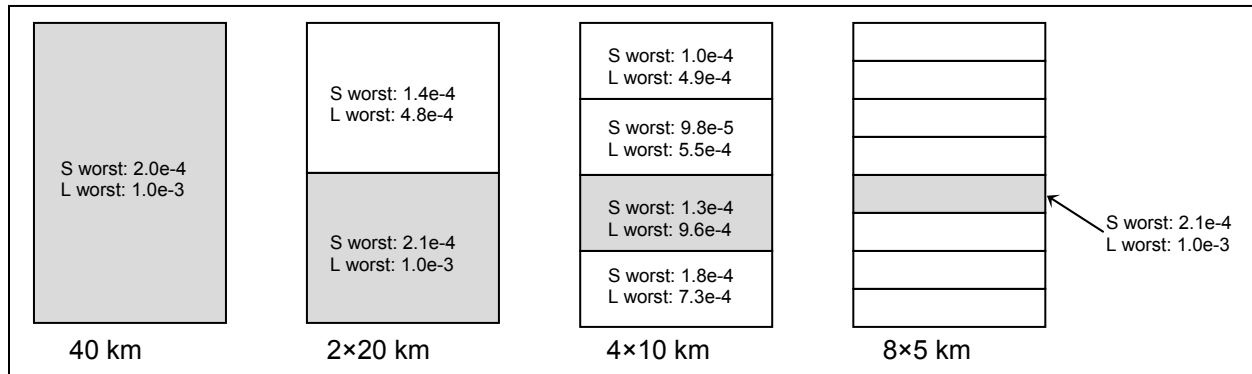
Using these scenes, two factors affecting RPC fit accuracy were analyzed: the strip length and the height range used for the 3D point grid (the effect of differing view geometry is considered indirectly through the use of multiple images). The procedure and results for these two tests are described in the following sections.

## **EFFECT OF STRIP LENGTH**

As mentioned in the introduction, the rational polynomial camera model used for IKONOS images uses cubic polynomials; these polynomials include terms up to the 3<sup>rd</sup> power in latitude, longitude, and height. This results in a set of 78 polynomial coefficients (39 each for sample and line) defining the ground-to-image geometry for Geo images.

Since the number of polynomial terms is the same regardless of image length, it is natural to expect a dropoff in accuracy for longer image strips. A test was thus devised to illustrate the effect of increasing strip length on RPC accuracy.

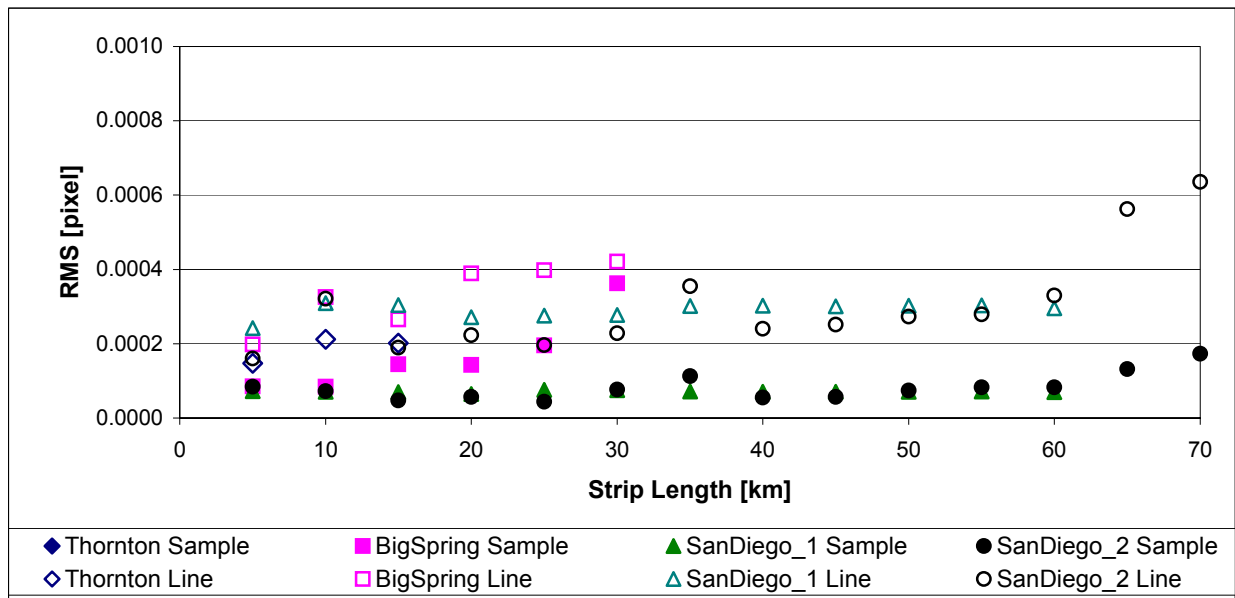
For each test scene, a 3D point grid and associated set of RPCs were computed for a number of different strip lengths, using the algorithm outlined in the previous section. If the strip length was less than the length of the scene, RPC estimation and analysis was repeated for as many segments as would fit into the scene, as depicted in Figure 1. For each segment of each scene, the least squares observation residuals were computed and used to determine RMS and worst-case error for sample and line. The worst result (i.e., the segment giving the highest overall worst-case sample or line residual error) for a given strip length was then used for further analysis.



**Figure 1.** For each strip length, the worst result is chosen from the different strips tested in the scene.

In all cases, the density of the 3D point grid generated via the IKONOS physical camera model was 200 pixels in line and sample, and 200 meters in height. This is dense enough so that grid density is not a factor affecting the fit accuracy; it also yields a very high redundancy in the equation system, making it valid to use observation residuals rather than independent check points for the accuracy assessment. The height range used for the tests was 2000 meters (ranging from zero to 2000 meters above the ellipsoid), a value suitable for areas of moderate terrain variation. The rectification height was chosen at the center of the height range, at 1000 meters.

Figure 2 presents the RMS errors for sample and line at different strip lengths, while Figure 3 shows the worst-case errors. In all cases, it can be seen that the results are very good; the highest RMS error, less than 0.001 pixel, is found in the line coordinate of the longest strip (SanDiego\_2 at a strip length of 70 km) while the worst-case error is smaller than 0.003 pixel. The results of this test are somewhat better than those predicted by Grodecki and Dial (2001), who gave an RPC accuracy value of 0.01 pixel RMS and 0.04 pixel worst case. However, their tests took into account virtually all imaging scenarios possible while this test is based only on a sample of typical IKONOS scenes.



**Figure 2.** RMS error vs. strip length

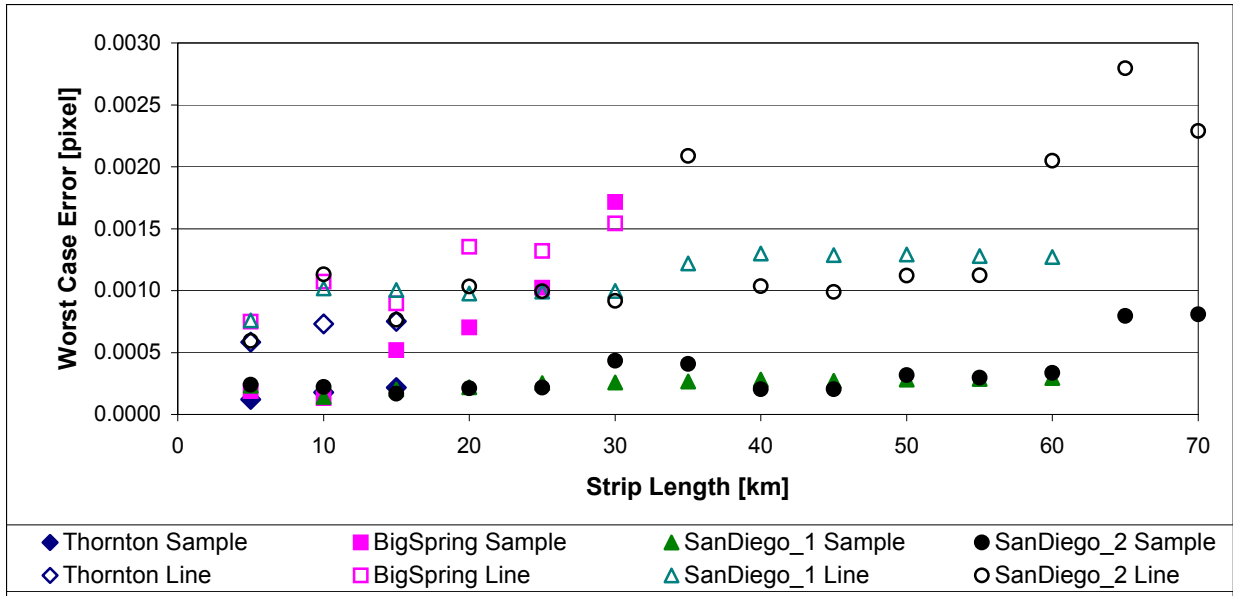


Figure 3. Worst-case error vs. strip length

From the plots, it can be seen that RMS and worst-case error grow only slightly with strip length. Surprisingly, the largest rate of error growth occurs with the BigSpring scene, even though it was captured from a high elevation angle. As seen in Table 1, however, this scene was captured in ‘forward scan’ mode; the scan azimuth of 0 degrees indicates that the scene was scanned from south to north, opposite to the satellite orbital motion. Forward scan scenes have a greater variability in ground-to-image geometry than do reverse scan scenes (Grodecki *et al.*, 2003), so this could be the cause of the observed results. Further work on this topic should include some longer scenes collected in forward scan mode so that this effect can be studied in more detail.

The longer SanDiego\_1 and SanDiego\_2 scenes, which were scanned from North to South, show very little variation with strip length. The SanDiego\_2 scene shows an increase in RMS and worst case error at strip lengths above 60 kilometers, but in any case the fit errors are so small as to be negligible for photogrammetric work with this imagery. From these results, it can be seen that strip lengths up to 70 kilometers are easily modeled by rational polynomials. Test data for strip lengths of greater than 100 kilometers will be needed in order to determine the point at which RPCs can no longer capture the full range of view geometry.

An examination of the full set of observation residuals provides some insight into the characteristics of the least squares fitting process. Figure 4 presents sample residuals from the 70 km SanDiego\_2 dataset, plotted with respect to latitude (this is practically the same as plotting with respect to time, since the scan took place in the North-South direction). Residuals from four of the height planes used in the 3D point grid are shown; it can be seen that the magnitude and variability of the residual errors increase steadily as distance from the rectification height increases.

The same pattern is evident in the plot of line residuals shown in Figure 5. Some of the same features can be seen in both residual plots, indicating that they are most likely due to the unmodeled effect of high-frequency changes in the line of sight during image capture.

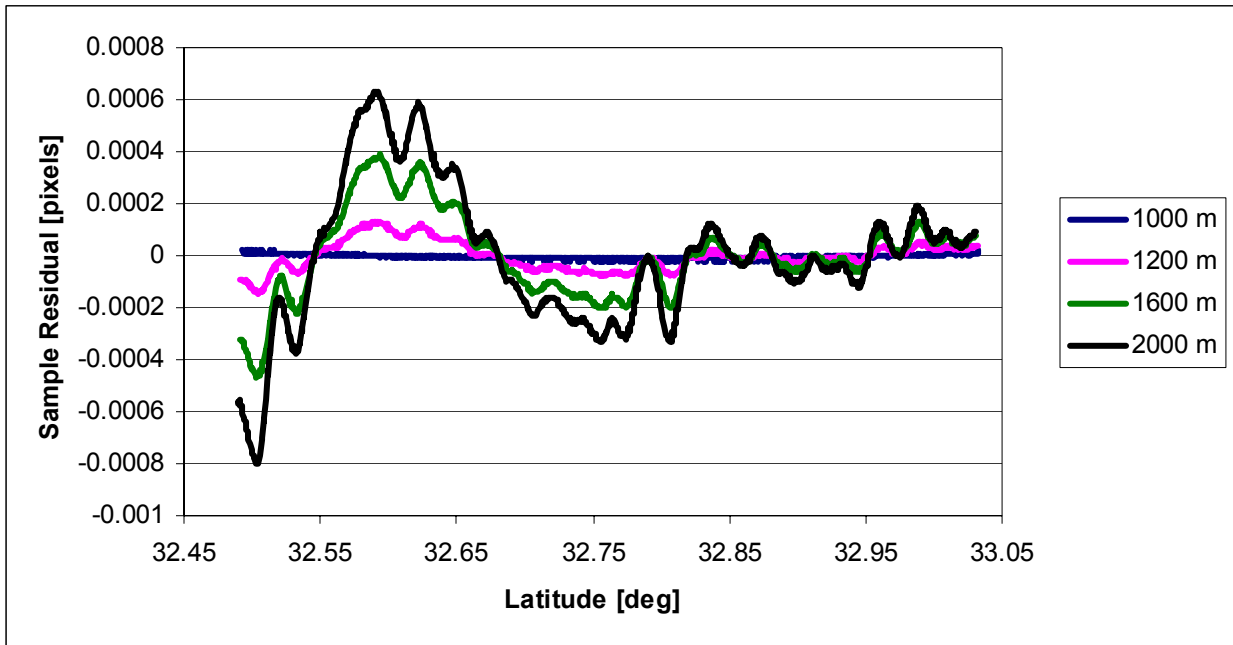


Figure 4. Sample residuals vs. latitude, SanDiego\_2 scene

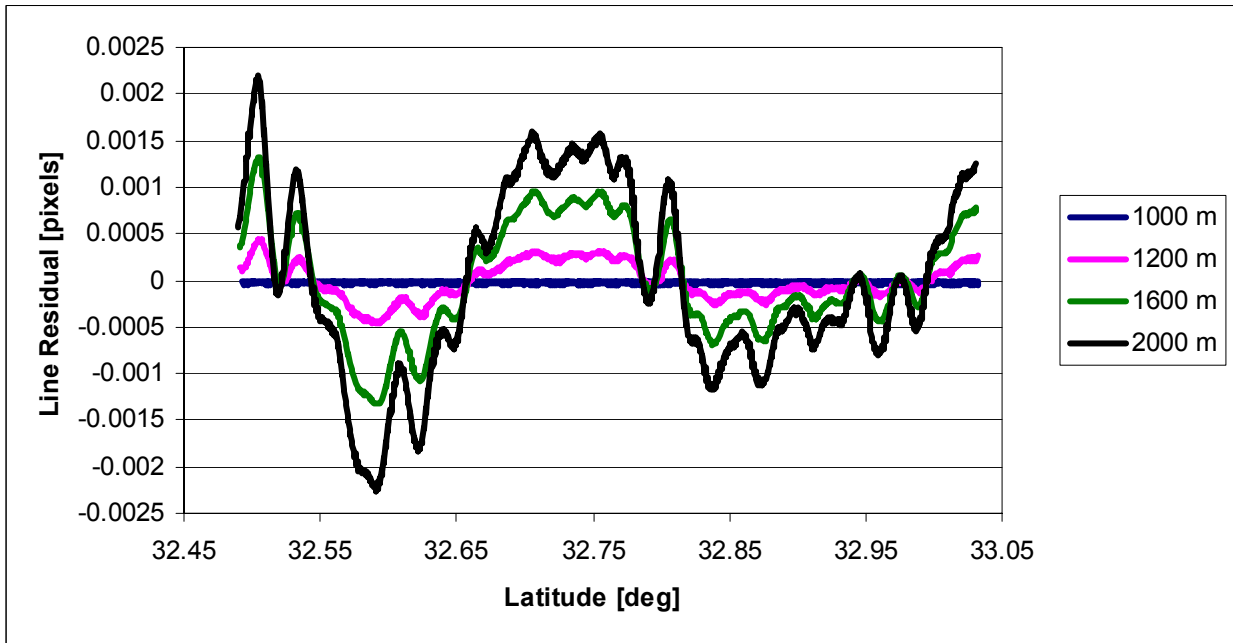
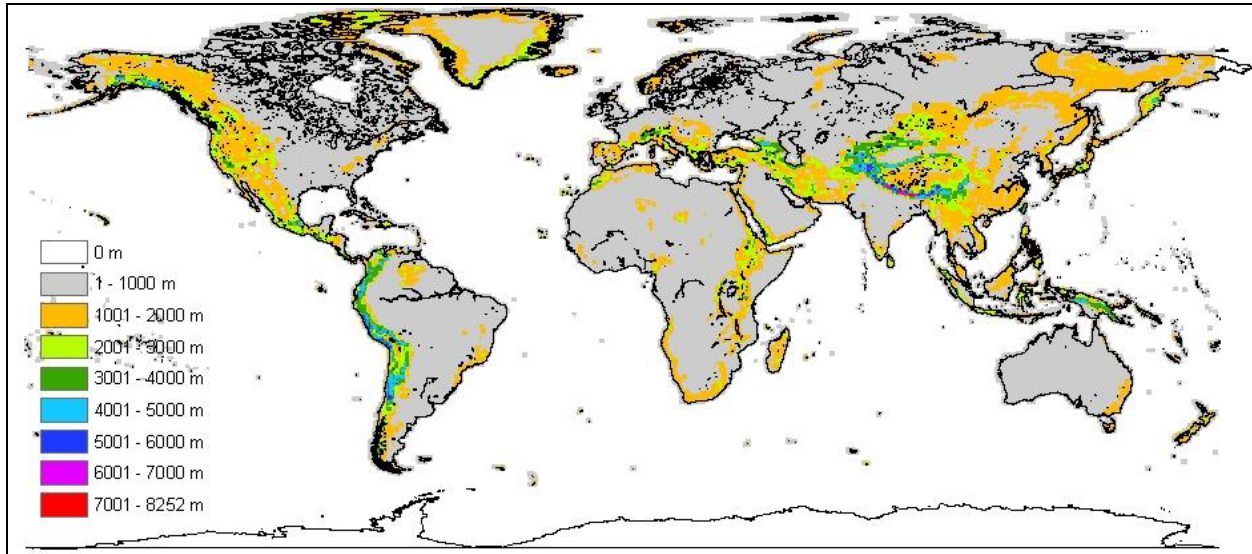


Figure 5. Line residuals vs. latitude, SanDiego\_2 scene

### EFFECT OF HEIGHT RANGE

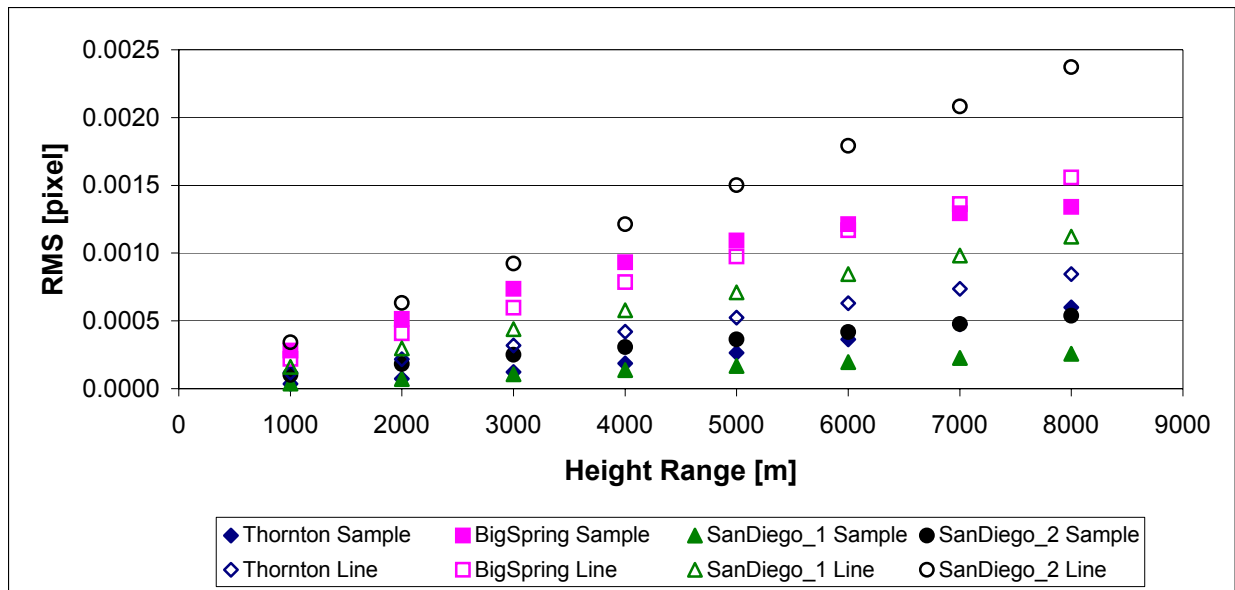
The tests shown in the previous section have demonstrated that RPC fit accuracy is very good for the scenes tested, using strip lengths of up to 70 kilometers. Unmodeled high-frequency variations in view geometry are evident in the residual errors, but they are on the order of millimeters and therefore negligible for all practical purposes.

The height range used for the tests of strip length was 2000 meters; for much of the world, this is more than enough to accommodate the height variation expected over an image. This can be confirmed by examination of Figure 6, which shows the maximum height range covered by a 100 km image strip collected anywhere in the world. However, it can also be seen that there are many regions in the world that cover a height range larger than 2000 meters, most notably the Andes and the Himalayas. At one point in the Himalayas, a 100 km long strip could cover as much as 8000 meters of terrain height variation.

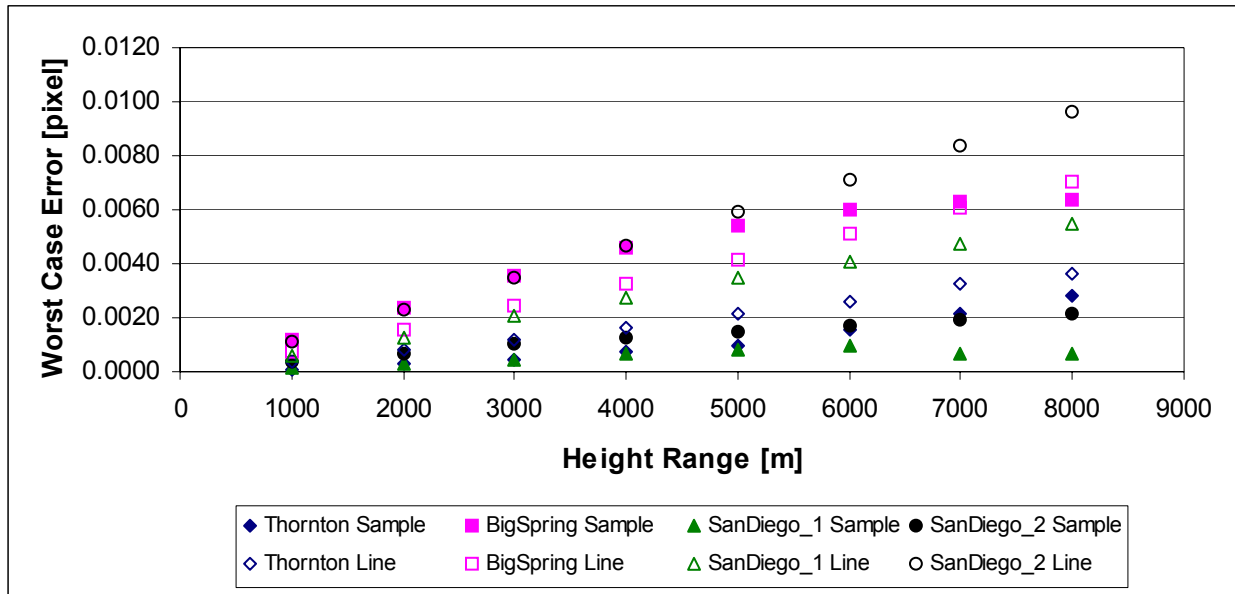


**Figure 6.** Maximum terrain height variation covered by 100 km scene

As with component length, it is natural to expect a dropoff in accuracy as the height range increases; in fact, Figure 4 and Figure 5 clearly showed that errors increase with distance from the rectification height. In order to test the effect of height range on RPC fit accuracy, the height range of the 3D point grid used for RPC estimation was varied from 1000 to 8000 meters. For each height range tested, RPCs were estimated for the full image length. RMS and worst-case errors were then computed for sample and line. Figure 7 demonstrates the relationship between height range and RMS error, while Figure 8 shows the worst-case errors.



**Figure 7.** RMS vs. height range  
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**Figure 8.** Worst-case error vs. height range

It can be seen that RPC fit accuracy decreases linearly with height range. However, it is obvious that large height ranges can be effectively modeled: at a height range of 8000 meters, sufficient for the most mountainous terrain on Earth, the worst-case error is less than 1 centimeter. This supports the finding of Fraser and Hanley (2004) who report that terrain variation has no noticeable effect on the accuracy of IKONOS RPCs.

## CONCLUSIONS

RPC accuracy has been investigated for a number of typical IKONOS scenes, ranging in length from 19 kilometers to 71 kilometers. Two main factors affecting RPC accuracy have been examined: image length and height range. It has been found that, for the images tested, RPC accuracy decreases only slightly with image length, reaching a maximum of less than 1 mm RMS and 3 mm worst-case for a 70 kilometer strip. The RPC accuracy for images scanned from South to North appear to worsen more quickly with length than those scanned from North to South; this is to be expected because the ground-to-image geometry changes much more quickly in the former case. However, the longest South-to-North strip used was only 30 kilometers long, so a larger dataset should be used for further work on this topic.

Investigation of the effect of height range on accuracy has shown that RPCs are valid for extremely mountainous regions; height ranges up to 8000 meters were tested, resulting in a worst-case error of less than 1 centimeter for a 70 kilometer strip. Thus, for all practical purposes it can be said that there is no loss of accuracy associated with the use of the rational polynomial camera model for these images.

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