

RPC REPLACEMENT CAMERA MODELS

Gene Dial^a, Jacek Grodecki^a

^aSpace Imaging, 12076 Grant St., Thornton CO 80241 USA – (gdial, jgrodecki)@spaceimaging.com

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ABSTRACT:

The increasing variety of digital aerial cameras and high-resolution satellite imaging systems creates a need for replacement camera models, i.e., a generic set of equations capable of accurately describing the object-to-imaging mapping of a variety of camera systems with a change of coefficient data. The Rational Polynomial Coefficient (RPC) camera model is simply the ratio of two cubic functions of latitude, longitude, and height that provide the conjugate image coordinate. The affine model and the frame camera DLT models are subsets of RPC models. RPC models have traditionally been used for rectification and feature extraction and have recently been extended to block adjustment. If a replacement model accurately represents the interior orientation of a camera, then the exterior orientation can be adjusted by transformation of the object space coordinate system. If the a-priori knowledge of exposure station position and angles permits a small angle approximation, then adjustment of the exterior orientation reduces to a simple bias in image space. RPC data accurately models IKONOS satellite images because of the smooth relationship between object and image space. The high accuracy of IKONOS, even without ground control, implies accurate a-priori so that block adjustment can be accomplished in image space. This paper focuses on the use of RPC replacement camera models for rectification, stereo feature extraction, and block adjustment in object and image space. The IKONOS satellite is used as an exemplar of the theory presented. RPC models are equally applicable to a variety of imaging systems and so could become a standardized representation of their image geometry.

1. INTRODUCTION

This article explores the application of replacement camera models, specifically the RPC camera model. The purpose of a replacement model is to describe the object-to-image mapping with a set of generic equations so that different camera designs can be described by a change of coefficient values. Replacement camera models greatly reduce the effort to implement camera models for new sensor designs in commercial, off-the-shelf software (COTS). The COTS developer need only implement the generic equations once. COTS software users can then apply that implementation to any sensor for which the required coefficient values were available.

We begin by describing rigorous camera models and replacement camera models including the RPC model, the DLT model, and the affine model. Then we describe how to use RPC models for orthorectification, for stereo extraction, and for block adjustment. Alternative block adjustment approaches are object space adjustment and image space adjustment. Object space adjustment is valid for any camera with known interior orientation. If the a-priori exterior orientation has small angular and scale errors, then a simpler, image space adjustment becomes valid. IKONOS satellite imagery is used as an example of a sensor with well-calibrated interior orientation and accurate a-priori exterior orientation permitting image space adjustment of sensor orientation.

RPC models have been used for many years as “fast” sensor models for rectification and stereo extraction. The NITF image format includes RPC (there called Rapid Positioning Coefficient) data for sensor orientation (NGA, 2000). Application of RPC’s to rectification and stereo extraction is described in (Tao & Hu, 2001). A technique for block adjustment of imagery described by RPC data was published in (Grodecki & Dial, 2003). Experimental verifications of RPC

accuracy include (Ager, 2003) and (Fraser, 2003). Application of RPC models to multi-sensor processing is described in (Tao & Hu, 2004).

2. RIGOROUS CAMERA MODEL

A camera model accurately maps object coordinates to image coordinates. Figure 1 shows a camera viewing an (X, Y, Z) object (ground) space with rays to an (x,y) image space. The camera model is separated into interior and exterior orientation. Interior orientation describes everything inside the camera such as optics and mechanical alignments. Exterior orientation is the position and attitude (view angle) during image exposure. Later we will describe an RPC model of the entire ground to image relationship in a single computation.

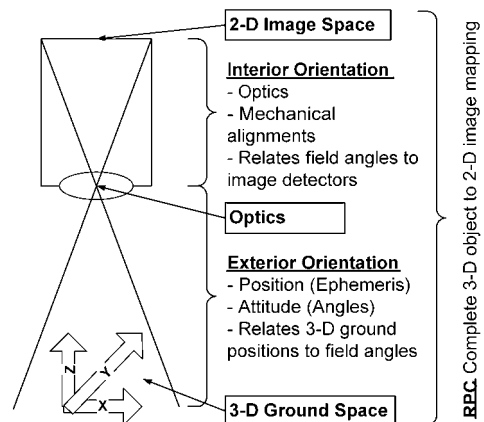


Figure 1. Interior and Exterior orientation of Rigorous Camera Model and RPC model of Complete 3D Object Coordinate to 2D Image Coordinate Mapping.

The interior orientation of a high-quality, solid-state, digital camera system should be geometrically stable across a broad range of operating conditions for long periods of time. Then the camera can be carefully calibrated in a laboratory or test range environment and that calibration can be applied to subsequent imaging operations. IKONOS interior orientation errors are less than one pixel worst case (Grodecki & Dial, 2002).

Consideration of image scale is intermediate between interior and exterior orientation because it is the ratio of the object distance to the focal length. Object distance is associated with exterior orientation and focal length with interior orientation. Scale errors for IKONOS are less than 50 parts per million (Dial & Grodecki, 2002) due to accurate ephemeris knowledge and focal length calibration.

High-resolution satellite imaging systems include sensors to measure exterior orientation. Star trackers and gyros measure the attitude angles of the satellite. GPS receivers measure the position of the satellite. Ground software integrates the sensor data to produce highly accurate exterior orientation data. For example, the IKONOS accuracy without ground control is about four meters RMS (Dial & Grodecki, 2002). Exterior orientation is thus sufficiently accurate for mapping at 1:10,000 scale without requiring ground control. Adjustment with ground control can be done by linear perturbation of the exterior orientation rather than a large angle adjustment because of the accurate a-priori exterior orientation provided by the satellite sensors.

While early aerial camera systems lacked exterior orientation sensors, increasing use of airborne GPS and IMU sensors is improving aerial systems abilities for direct georeferencing.

3. REPLACEMENT CAMERA MODELS

The purpose of a replacement camera model is to provide a simple, generic set of equations to accurately represent the ground to image relationship of the physical camera. We might write that relationship as $(x, y) = \mathbf{R}(P_G)$ where $\mathbf{R}(\cdot)$ is the replacement camera model function, (x, y) is an image coordinate, and P_G is a ground coordinate. Ideally, one set of equations, with different coefficient values, could model images from multiple camera designs. This greatly simplifies the task for COTS software vendors—they can implement one set of equations and then process imagery from different camera systems.

A replacement camera model must not only model the ground-to-image relationship accurately, but must also perform the tasks normally performed with a physical camera model. It must also support orthorectification, stereo feature extraction, DEM extraction, and block adjustment. In the sections that follow we will describe the RPC camera models of high-resolution satellite and frame cameras and how to use RPC models for orthorectification, 3D feature extraction, and block adjustment.

4. RPC CAMERA MODEL

The RPC equations constitute a replacement camera model in that they are a generic set of equations that map object coordinates into image coordinates, as shown in Figure 2, for a variety of sensor systems. The RPC coefficients describe a

particular image from a particular imaging system. The RPC coefficients are used in the RPC equations to calculate an image (sample, line) coordinate from an object (longitude, latitude, height) coordinate.

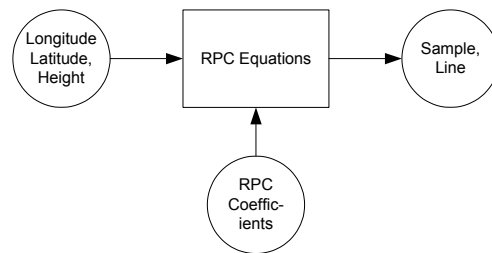


Figure 2. RPC Equations Map Object Coordinate to Image Coordinate using RPC Coefficients

The RPC coefficients and object coordinates are input. The equations begin by scaling the object space coordinates to a range of ± 1 . Then the ratio of two cubic functions is calculated for sample and line. Finally we denormalize the result to sample and line. Complete equations are shown below.

$$x = \frac{X - x_0}{x_s}$$

$$y = \frac{Y - y_0}{y_s} \tag{1}$$

$$z = \frac{Z - z_0}{z_s}$$

$$u = \frac{\sum_{i=1}^{20} C_{Si} \cdot \rho_i(x, y, z)}{\sum_{i=1}^{20} D_{Si} \cdot \rho_i(x, y, z)} \tag{2}$$

$$v = \frac{\sum_{i=1}^{20} C_{Li} \cdot \rho_i(x, y, z)}{\sum_{i=1}^{20} D_{Li} \cdot \rho_i(x, y, z)}$$

where

$$\sum_{i=1}^{20} C_i \cdot \rho_i(x, y, z) = C_1 + C_2 \cdot x + C_3 \cdot y + C_4 \cdot z$$

$$+ C_5 \cdot x \cdot y + C_6 \cdot x \cdot z + C_7 \cdot y \cdot z + C_8 \cdot x^2$$

$$+ C_9 \cdot y^2 + C_{10} \cdot z^2 + C_{11} \cdot y \cdot x \cdot z + C_{12} \cdot x^3 \tag{2}$$

$$+ C_{13} \cdot x \cdot y^2 + C_{14} \cdot x \cdot z^2 + C_{15} \cdot x^2 \cdot y$$

$$+ C_{16} \cdot y^3 + C_{17} \cdot y \cdot z^2 + C_{18} \cdot x^2 \cdot z$$

$$+ C_{19} \cdot y^2 \cdot z + C_{20} \cdot z^3$$

$$S = u \cdot S_s + S_0 \tag{4}$$

$$L = v \cdot L_s + L_0$$

There in less than half a page is a camera model capable of describing the entire complexity of a high-resolution satellite camera image with 80 coefficients and 10 normalizing parameters.

4.1 RPC Accuracy for High Resolution Satellite imagery

Comparisons of RPC models fit to IKONOS rigorous camera model have shown negligible, less than 0.1 pixel, residual errors (Lutes 2004, Grodecki 2003).

RPC coefficients are available for IKONOS, QuickBird, and OrbView3 imagery from Space Imaging, Digital Globe and OrbImage respectively.

4.2 Central Perspective Cameras

If only the linear terms are retained in the numerators and denominators of the RPC equations, then one obtains the DLT equations used to describe central perspective cameras:

$$\begin{aligned} x &= \frac{C_{X1} + C_{X2} X + C_{X3} Y + C_{X4} Z}{D_{X1} + D_{X2} X + D_{X3} Y + D_{X4} Z} \\ y &= \frac{C_{Y1} + C_{Y2} X + C_{Y3} Y + C_{Y4} Z}{D_{Y1} + D_{Y2} X + D_{Y3} Y + D_{Y4} Z} \end{aligned} \quad (5)$$

where (x, y) = focal plane coordinate, (X, Y, Z) = ground coordinate, and $\{C_{Xi}, D_{Xi}, C_{Yi}, D_{Yi}\}$ = DLT coefficients defining the camera object-to-image relationship. Frame cameras can thus be exactly represented with RPC equations in rectilinear coordinates. Higher order coefficients can accommodate optical distortions or non-linear coordinate systems.

4.3 Affine Model

Some authors have attempted to use an affine transformation to model satellite image geometry. If (x,y) is an image coordinate and (X, Y, Z) is an object coordinate then the affine transformation can be written

$$\begin{aligned} x &= c_1 + c_2 X + c_3 Y + c_4 Z \\ y &= c_5 + c_6 X + c_7 Y + c_8 Z \end{aligned} \quad (6)$$

An affine could precisely represent a satellite image if the view angle were perfectly constant throughout the image. The affine model breaks down to the extent view angle is not constant throughout the image. View angle changes across the field of view of high-resolution satellite image are small (e.g. IKONOS has a 0.9° FOV), view angle can change appreciably due to the pushbroom scanning motion of the satellite, particularly if the image strip is long or if it is not parallel to the satellite motion. Although simple affine models have been shown to perform reasonably well in some cases, difficulties have been encountered in others, suggesting that the affine model is not sufficiently general to describe present day imaging systems.

4.4 Orthorectification

RPC equations describe ground-to-image geometry as shown in Figure 3. An algorithm for orthorectification of an image with RPC camera model is shown in Figure 4.

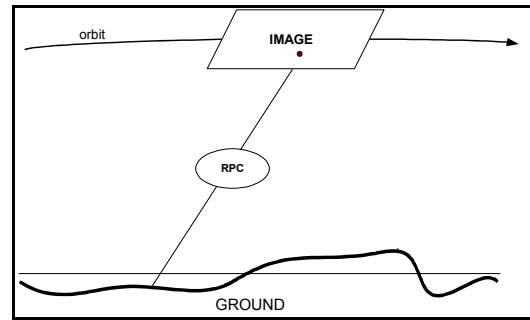


Figure 3. RPC Image Geometry for Orthorectification

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For each pixel P in orthorectified image
Calculate the map (X, Y) coordinate of P
Convert (X, Y) to (λ, φ)
Interpolate DEM at (λ, φ) to obtain H
Add geoid height: h = H + N
Calculate image coord with RPC eqns:
(L, S) = RPC(λ, φ, h)
Interpolate the input image at (L, S)
to determine the DN value of P
Next pixel P
    
```

Figure 4. RPC Orthorectification Algorithm

Orthorectification can be accomplished with an RPC camera model and a DEM using the algorithm above. Symbol P identifies a pixel in the output, orthorectified image, with map (easting, northing) coordinate (X, Y) , longitude λ , latitude ϕ , orthometric height H, geoid height N, and ellipsoidal height h. Symbols (L, S) identify the (line, sample) of the input image to be resampled into the output, orthorectified image. Typically a DEM provides height above geoid, so the geoid separation N is added to give the height above ellipsoid, h, required for the RPC equations.

4.5 Stereo extraction

The stereo extraction problem is illustrated in Figure 5. Two images are each described by their own sets of RPC coefficients. The task is to calculate the object space coordinate (ϕ, λ, h) from conjugate image coordinate measurements (L_1, S_1) and (L_2, S_2) . The problem is over-determined with three unknowns (ϕ, λ, h) to be calculated from the four RPC equations for (L_1, S_1) and (L_2, S_2) . Details of this calculation may be found in (Grodecki *et al* 2004).

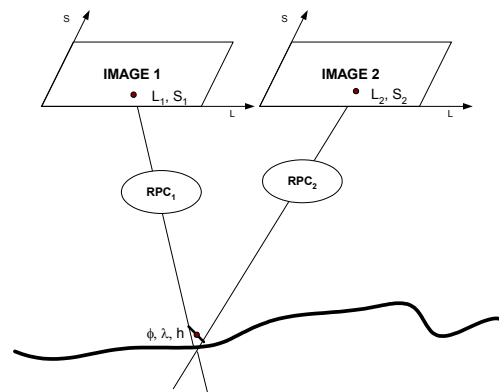


Figure 5. Stereo Extraction by RPC

5. RPC BLOCK ADJUSTMENT

Block adjustment is the adjustment of orientation parameters using available ground control, tie-point, and a-priori data. The interior orientation of a well-formed camera system is already known from calibration and so does not require adjustment. Ground control point coordinates do not require further adjustment if surveyed to sufficient accuracy. So we will discuss adjusting the exterior orientation of imagery described by replacement models in general and RPC models in particular.

5.1 Object space adjustment

Consider the situation shown in figure 6. A camera takes an image described by RPC or some other replacement camera model. The RPC model correctly describes the interior orientation of the image but the exterior orientation requires adjustment. We could describe that adjustment as a coordinate transformation between the adjusted coordinate system and the RPC coordinate system.

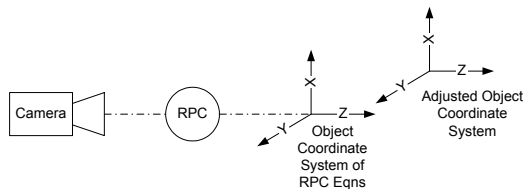


Figure 6. Adjusting Exterior Orientation by Transformation of RPC Object Coordinate System

Let P_G be a ground point in the object space of the RPC equations and P_G' be a ground point in the adjusted coordinate space. A transformation from P_G to P_G' could be written $P_G' = R_z(A_z)R_y(A_y)R_x(A_x)(P_G - P_O)$ where P_O is the offset and $R_x(A_x)$, $R_y(A_y)$, and $R_z(A_z)$ are rotations about their respective axes by angles A_x , A_y , and A_z . On substituting P_G' into the replacement model equation we obtain the camera equation with adjustable exterior orientation parameters P_O , A_x , A_y , and A_z :

$$(x, y) = \mathbf{R}(R_z(A_z)R_y(A_y)R_x(A_x)(P_G - P_O)) \quad (7)$$

Thus a replacement model that accurately represents the interior orientation of a camera can be exactly adjusted to change exterior orientation.

5.2 Image space adjustment

If the offsets and rotations are small, then we can expand equation 7 in a Taylor series, keeping only the bias term:

$$(x, y) = \mathbf{R}(P_G) + (Bx, By) \quad (8)$$

In this bias-compensated RPC model, Bx absorbs the first-order effect of camera displacements along the x-axis and rotations about the y-axis while By absorbs the first-order effects of displacements along the y-axis and rotations about the x-axis. Higher order terms are not needed if the field of view is narrow and the parameter errors are small. Solution for the RPC bias terms in (8) is particularly easy. Simply set the biases to the average image error to remove the effect of exterior orientation

errors. The equivalence of small translations perpendicular to the optic axis and small rotations about axes perpendicular to the optic axis is discussed in (Dial & Grodecki, 2004).

The image space adjustment of RPC replacement models does not adjust for rotation about the optic axis nor does it adjust for scale. The effect of rotation about the optic axis is small for a narrow field camera with strong a-priori. For example, if the half field-of-view θ is 0.01 radians, the rotation error A_z about the optic axis is 10^{-5} radians (about 2 arc-sec), and the height h is 500 km, then the effect of the rotation error is $\theta A_z h = 0.05$ meters and not perceptible at today's resolutions. In general, the effect of rotation about the optic axis is negligible if $\theta A_z h \ll \text{GSD}$. The effect of scale errors can also be negligible if the focal length and object distance are accurately known. For IKONOS, the focal length is accurately calibrated and orbital height is only uncertain to about one meter RMS so the ratio of height uncertainty to height is only 1.5 parts per million. Thus yaw rotations and scale errors are negligible for IKONOS and any similarly accurate systems.

5.3 Covariance Calculation

If system accuracy is expressed in terms of image-space biases used to weight the observation equations, then the estimation error covariance can be calculated as a by-product of the least-squares solution. For example, IKONOS biases are less than 4 m 1- σ per axis and relative errors are less than 50 ppm 1- σ (Dial and Grodecki, 2002). Covariance calculation from sensor orientation uncertainty is discussed in (Theiss, Mikhail, & Corsetti, 2004). An earlier discussion of estimation covariance from RPC models can be found in (Downman & Dolloff 2000).

6. SUMMARY & CONCLUSIONS

A simple, standardized mathematical formulation capable of accurately representing the image geometry of a variety of camera systems would facilitate software implementation and multi-sensor processing. Because the RPC model accurately fits the object-image geometry of a variety of camera systems and is computationally efficient, it has traditionally been applied to post-triangulation applications such as rectification and feature extraction. Replacement camera models, including RPC models, can be adjusted for exterior orientation without loss of accuracy and can be adjusted in image space if the a-priori orientation permits small-angle approximations. RPC model data is available for imagery from the IKONOS, QuickBird, and OrbImage satellites. Because of their utility in photogrammetric operations (rectification, extraction, and block adjustment) and their generality representing diverse cameras, including satellite and frame cameras, RPC models are recommended to the photogrammetric community as a replacement camera model.

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